Observability-Aware Trajectory Optimization for Self-Calibration with Application to UAVs

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***Abstract*—In this work, we develop an observability-aware trajectory optimization framework for nonlinear systems that produces trajectories well suited for self-calibration. Our method reasons about the quality of observability while respecting system dynamics and motion constraints. Experiments on a simulated quadrotor with a IMU-GPS sensor suite demonstrate the benefits of our method compared to a covariance-based approach and multiple heuristic approaches. Our method is considerably faster than the covariance-based approach and achieves better results than any other approach in the self-calibration task.**

I. Introduction

State estimation is a core capability for autonomous robots, serving as the foundation for control, higher-level planning, and perception. In addition to the states directly used for system control, such as position, velocity, and attitude, recent work also estimates internal states that calibrate the sensor suite [8]. These *self-calibration states* include extrinsic parameters such as the position of one sensor with respect to another, and intrinsic parameters such as measurement bias.

Estimating these states online, rather than in an offline calibration process, makes the system simpler and more robust against parameter changes, e.g. from collisions. However, it comes at a cost: the dimensionality of the state vector increases while the number of measurements remains unchanged. This often leads to a scenario where a "natural" trajectory, such as one from an energy-minimizing planner, does not excite the system sufficiently to keep all states observable.

In this work, we present a framework that optimizes trajectories for self-calibration. We develop a cost function that explicitly addresses the quality of observability of system states. Our method takes into account motion constraints and yields an optimal trajectory for fast convergence of the self-calibration states. The presented theory applies to any nonlinear system and is not specific to any state estimator. While past approaches have focused on analyzing the environment to compute *where* to move to obtain informative measurements for state estimation [1, 2, 3, 6], we assume the presence of accurate measurements and focus on *how* to move to generate motions that render the full state space observable.

We evaluate our method with several experiments on a simulated Unmanned Aerial Vehicle (UAV) with IMU-GPS sensor suite. An example of the performance of the self-calibration framework is presented in Fig. 1, where an optimized trajectory outperforms common calibration heuristics in terms of speed and accuracy of state convergence.

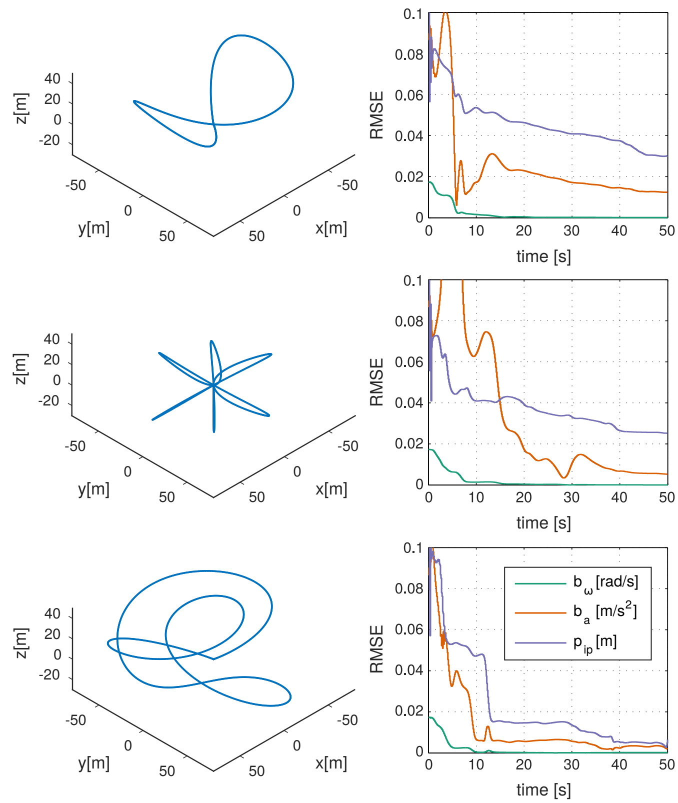

Fig. 1. Root Mean Squared Error (RMSE) convergence of EKF self-calibration (accelerometer and gyroscope biases $\mathbf{b}_a$, $\mathbf{b}_\omega$ and position of the GPS sensor $\mathbf{p}_i^p$) state estimates for a) figure-eight trajectory, b) star trajectory, c) optimal trajectory from our method. We introduced additional yaw motion for a) and b) trajectories in order to improve state estimation of these heuristics.

II. Nonlinear Observability Analysis

In control theory, the observability of a system is defined as the possibility to compute the initial state of the system given a sequence of inputs and measurements. Observability of a nonlinear system along a specific trajectory can be determined from the rank of the observability matrix. However, this is a binary test and does not quantify how *well* observable the system is. This limits its utility for gradient-based methods.

Krener and Ide [4] introduced continuous measures of observability based on the *local observability Gramian*. Unfortunately, the local observability Gramian is difficult to compute for an arbitrary nonlinear system. To deal with this problem, [4] then introduced the *empirical local observability Gramian* as a numerical approximation. However, this approximation presents problems for our quadrotor application. In the standard formulation of quadrotor state estimation, IMU odometry is treated as a control rather than a measurement. Thus, the empirical local observability Gramian can only capture the effect of IMU bias implicitly via double integration. This introduces numerical errors that cannot be distinguished from the true effect of the IMU biases.

To overcome this limitation, we introduce a novel approximation of the local observability Gramian that analytically captures input-output dependencies not visible in the sensor model. We achieve this property by incorporating higher order Lie derivatives from the nonlinear observability matrix. Intuitively, at each time step, we evaluate the local Taylor expansion of the sensor model at a fixed time horizon to approximate how the system measurements change with respect to a small perturbation of the self-calibration states. We use this approximation to estimate the local observability Gramian which is integrated over the entire trajectory.

III. Trajectory Representation and Optimization

We represent a trajectory by a piecewise polynomial in the system's minimal set of differentially flat variables. As shown in Müller and Sukhatme [7], with an appropriately high polynomial degree, piecewise polynomial trajectory planning forms an underdetermined linear system. Therefore, we compute an initial solution with the Moore-Penrose pseudoinverse, and use the remaining null space of the system as the optimization space. Optimizing in the null space transforms the problem into an unconstrained optimization. However, we must still add constraints to keep the trajectories physically plausible.

We perform a local optimization over this space using Sequential Quadratic Programming. The physical limitations of the system, such as maximum motor torques, are expressed as nonlinear inequality constraints using the Barrier method [5]. The cost function is the smallest singular value of our novel local observability Gramian approximation.

IV. Example Application to UAV with IMU-GPS

We demonstrate the presented theory on a simulated quadrotor with a 3-DoF position sensor (e.g. GPS) and a 6-DoF inertial measurement unit (IMU). This is a simple, widely popular sensor suite, but it presents a challenging self-calibration task, as there is limited intuition for what kind of trajectory would make the states well observable.

The simulated quadrotor performs state estimation with an indirect Extended Kalman Filter. The filter state includes position, velocity, orientation quaternion, IMU-GPS offset, and biases of the accelerometer and gyroscope. The differentially flat variables used for trajectory representation are x, y, z

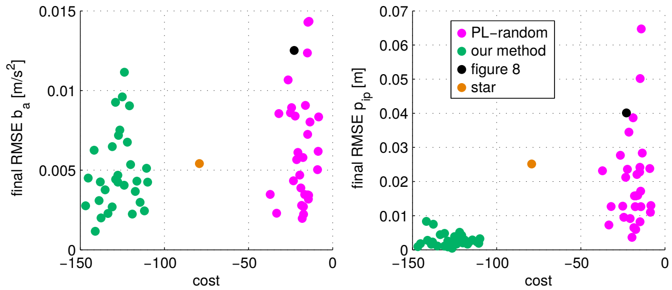

Fig. 2. Self-calibration task: final RMSE values for the accelerometer bias $\mathbf{b}_a$ and the GPS position in the IMU frame $\mathbf{p}_i^p$ obtained using optimization (green) and 3 different heuristics: star and figure eight trajectories from Fig. 1 and randomly sampled trajectories that are close of the physical limits of the system (magenta).

position and yaw. To ensure that trajectories are physically plausible we place inequality constraints on 3 entities: the thrust-to-weight ratio, angular velocity, and angular acceleration. IMU-GPS offset and IMU biases are set to fixed realistic nonzero values, but the EKF is initialized with the belief that all self-calibration states are zero. Thus, a bad self-calibration trajectory will fail to converge the state estimate of the system.

Fig. 2 shows optimization results. We generated random closed-loop self-calibration trajectories (*PL-random*) that are close to the system's physical limits. We then used each random trajectory as an initial condition for nonlinear optimization to produce an optimized trajectory (*our method*). We also compared against common heuristic trajectories (*star* and *figure 8*). The left plot shows results from optimizing for the accelerometer bias $\mathbf{b}_a$, and the right for IMU-GPS offset $\mathbf{p}_i^p$. Initial tests showed that the gyroscope bias converges quickly for almost any trajectory, so did not include it in the evaluation. While the *star* trajectory and some of the *PL-random* trajectories perform well on $\mathbf{b}_a$, our approach outperforms all other methods on $\mathbf{p}_i^p$.

A baseline method for comparison is minimizing the trace of the EKF covariance estimate at the end of the trajectory. This method is computationally heavy, taking $\sim$80x longer in our Matlab implementation. It was thus infeasible to compute the covariance optimization for all 50 *PL-random* initial trajectories. Instead, we compared a single representative trajectory from each optimization framework, and found comparable results. We omit these results due to space constraints. We also omit an experimental evaluation on a waypoint navigation task, where our method was able to significantly improve the final state estimate compared to a minimum-snap trajectory.

V. Conclusion

Our initial work shows encouraging results on the simulated quadrotor with IMU-GPS sensor suite. Since a major advantage of the presented method is its generality, we plan to expand our work with experiments on other sensor suites, including a GPS-denied visual-inertial navigation system. Furthermore, we plan to compare more thoroughly against covariance-based optimization, and to test our approach on a real quadrotor and other robotic systems.

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